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REDUCED FIRST ZAGREB INDEX, REDUCED SECOND ZAGREB INDEX AND HYPER-ZAGREB INDEX OF FOUR NEW SUMS BASED ON TENSOR PRODUCT OF GRAPHS

Abhigyan Mahanta[*1], Jibonjyoti Buragohain[2] & Ankur Bharali[3]

[*1,2]Research Scholar, Department of Mathematics, Dibrugarh University, Assam-786004, India

[3]Assistant Professor, Department of Mathematics, Dibrugarh University, Assam-786004, India

Abstract

Topological indices have opened up doors for better studies in structural chemistry and biology. In this paper, we compute the Reduced first Zagreb index, Reduced second Zagreb index and hyper-Zagreb index of four new graph operations based on tensor product.

Key words: *Topological index, Graph operation, Zagreb indices and coindices.*

I. INTRODUCTION

We have only taken up graphs that are finite, simple, undirected and connected in this paper. The vertex set and edge set of a graph G have been denoted as $V(G)$ and $E(G)$ respectively. We use $\delta_G(u)$ to represent the degree of a vertex u in G.

The topological indices are numerical values which are linked to a graph structure. In modeling information of molecules in structural chemistry and biology, they have been found to be of sublime advantage. They aid in quantifying the respective graph's molecular structuring and branching pattern as a consequence of setting up a correlation of chemical structure with various physical, chemical and biological properties of the graph.

During the last few decades, topological indices have drawn the attention of researchers because of the relative ease of generation and the speed with which these computations can be completed. The computation of an index for various operations of graphs can greatly aid in determining the value of that index for complex graph structures. This throws light upon the importance of these indices.

Numerous topological indices have been proposed and studied over the years based on vertex-degree, eccentricity, distance, etc. One may refer to [2] for a deeper insight into some degree-based topological indices. However, only some of them have been found to have broader applications.

The Zagreb indices can be considered to be historically the first degree-based topological index which came into picture during the study of total *π-electron energy of alternant hydrocarbons* by *Gutman* and *Trinajstić* in 1972 [1]. But they were placed in the niche of topological indices much later. This delay was mostly due to their completely different purpose of utility.

We now state the formal definitions of some of these indices (descriptors) that we have used in our study.
The first Zagreb index, second Zagreb index [2], forgotten topological index [11] , the hyper-Zagreb index [12], the Reduced first Zagreb index [13] and Reduced second Zagreb index [14] of a graph are defined as follows.

$$M_1(G) = \sum_{u\in V(G)} d_G^2(u) = \sum_{u\in V(G)} \left(d_G(u) + d_G(u)\right),$$

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$$M_2(G)=\sum_{uv\in E(G)} d_G(u)d_G(v),$$

$$M_3(G)=\sum_{u\in V(G)} d_G^3(u)=\sum_{uv\in E(G)}\left(d_G^2(u)+d_G^2(v)\right),$$

$$HZ(G)=\sum_{uv\in E(G)}\left(d_G(u)+d_G(v)\right)^2,$$

$$RM_1(G)=\sum_{u\in V(G)}(d_G(u)-1)^2,$$

$$RM_2(G)=\sum_{uv\in E(G)}(d_G(u)-1)(d_G(v)-1).$$

The generalized Zagreb index of a graph *G*, denoted by $M_\alpha(G)$, is given as:

$$M_\alpha(G)=\sum_{u\in V(G)} d_G^\alpha(u)=\sum_{uv\in E(G)}\left(d_G^{\alpha-1}(u)+d_G^{\alpha-1}(v)\right),$$

where $\alpha\neq 0,1$.

Significant research works based on topological indices of graph operations have come to light during the last few decades. Eliasi and Taeri [3] determine the Wiener index four new sums of graphs. Sarkar, De and Pal [6] study the Zagreb indices of graphs based on new operations related to the join of graphs. Akhter, Imran [5] compute the forgotten topological index of four operations of graphs. Imran, Baby, Siddiqui and Shafiq [9] determine the lower and upper bounds for general Randić, general sum-connectivity and harmonic indices for tensor product of certain graph operations on graphs. Shwetha et al. [4] derives expressions for the harmonic index for the join, corona product, Cartesian product, composition and symmetric difference of graphs. Onagh [10] came up with the harmonic index of edge-semitotal graphs, total graphs and their related sums.

The tensor product of two graphs G_1 and G_2, denoted by $G_1\otimes G_2$, is a operation on graphs. The tensor product generates a graph with vertex set $(G_1)\times V(G_2)$, where "$\times$" denotes the Cartesian product of sets and two vertices (u_1,v_1) *and* (u_2,v_2) in $G_1\otimes G_2$ is connected if and only if u_1 is connected to u_2 and v_1 is connected to v_2.

In this paper, we redefine (for the sake of completeness of the paper) the new graph operation which was defined in [7] and establish explicit formulae for the Reduced first Zagreb index, Reduced second Zagreb index and the hyper-Zagreb index of the new operation. The rest of the paper is organized as follows: In section 2, we restate the new graph operation and some preliminary results are given. In section 3, expressions for the Reduced first Zagreb index, Reduced second Zagreb index and the hyper-Zagreb index of the new operation are computed. In section 4, conclusions are made.

II. NEW *F*-SUMS OF GRAPHS

Let $F=\{S,Q,R\ and\ T\}$, where $S,Q,R\ and\ T$ stands for the *Subdivision graph*, *Vertex-semitotal graph*, *Edge-semitotal graph* and *Total graph* respectively. Details of these four derived graphs may be found in [3]. For any two simple connected graphs G_1 and G_2, a new graph operation $G_1\otimes_F G_2$ has been defined in [7]. This

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operation is defined as: $G_1 \otimes_F G_2 = F(G_1) \otimes G_2$ i.e. $G_1 \otimes_F G_2$ is the tensor product of $F(G_1)$ and G_2. The vertex set and the edge set of the new operation are given as:

$$V(G_1 \otimes_F G_2) = V(G_1) \cup E(G_1) \cup V(G_2) \text{ and}$$

$$E(G_1 \otimes_F G_2) = \{(u_1,v_1)(u_2,v_2) \mid u_1u_2 \in E(F(G_1)) \text{and } v_1v_2 \in E(G_2)\}.$$

In this paper, we propose explicit expressions for the Reduced first Zagreb index, Reduced second Zagreb index and the hyper-Zagreb index for four new operations based on the Tensor product of graphs.

Lemma 2.1: [7] *Let G_1 and G_2 be two graphs. Then,*

i.

$$\delta_{G_1 \otimes_S G_2}(a,b) = \begin{cases} \delta_{G_1}(a)\delta_{G_2}(b) & if\ a \in V(G_1) \\ 2\delta_{G_2}(b) & if\ a \in V(S(G_1)) \setminus V(G_1). \end{cases}$$

ii.

$$\delta_{G_1 \otimes_R G_2}(a,b) = \begin{cases} 2\delta_{G_1}(a)\delta_{G_2}(b) & if\ a \in V(G_1) \\ 2\delta_{G_2}(b) & if\ a \in V(R(G_1)) \setminus V(G_1). \end{cases}$$

iii.

$$\delta_{G_1 \otimes_Q G_2}(a,b) = \begin{cases} \delta_{G_1}(a)\delta_{G_2}(b) & if\ a \in V(G_1) \\ \left(\delta_{G_1}(w) + \delta_{G_1}(w')\right)\delta_{G_2}(b) & if\ a \in V(Q(G_1)) \setminus V(G_1), \end{cases}$$

where in the second case a is inserted in $ww' \in E(G_1)$.

iv.

$$\delta_{G_1 \otimes_T G_2}(a,b) = \begin{cases} 2\delta_{G_1}(a)\delta_{G_2}(b) & if\ a \in V(G_1) \\ \left(\delta_{G_1}(w) + \delta_{G_1}(w')\right)\delta_{G_2}(b) & if\ a \in V(T(G_1)) \setminus V(G_1), \end{cases}$$

where in the second case a is inserted in $ww' \in E(G_1)$.

Theorem 2.2: [7] *Let G_1 and G_2 be two simple finite graphs with $|E(G_1)| = m_1$. Then, we have the following results for the first Zagreb index.*

a. $M_1(G_1 \otimes_S G_2) = M_1(G_1)M_1(G_2) + 4m_1M_1(G_2)$.
b. $M_1(G_1 \otimes_R G_2) = 4M_1(G_1)M_1(G_2) + 4m_1M_1(G_2)$.
c. $M_1(G_1 \otimes_Q G_2) = (M_1(G_1) + 2M_2(G_1) + M_3(G_1))M_1(G_2)$.
d. $M_1(G_1 \otimes_Q G_2) = (4M_1(G_1) + 2M_2(G_1) + M_3(G_1))M_1(G_2)$.

Theorem 2.3: [7] *Let G_1 and G_2 be simple connected undirected graphs. Then, the second Zagreb indices of $G_1 \otimes_F G_2$ are given as below:*

i. $M_2(G_1 \otimes_S G_2) = 4M_1(G_1)M_2(G_2)$.
ii. $M_2(G_1 \otimes_R G_2) = 8(M_1(G_1) + M_2(G_1))M_2(G_2)$.
iii.

$$M_2(G_1 \otimes_Q G_2) = [2M_2(G_1) + M_3(G_1) + M_4(G_1) + 2\sum_{w,w' \in V(G_1)} \gamma_{ww'}\delta_{G_1}(w)\delta_{G_1}(w')$$

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$$+2\sum_{w'\in V(G_1)}\left(\delta_{G_1}(w')\right)^2\sum_{w\in V(G_1)s.t.\ ww'\in E(G_1)}\delta_{G_1}(w)]M_2(G_2),$$

where $\gamma_{ww'}$ *is the number of common neighbors of* w *and* w' *in* G_1.

iv.

$$M_2(G_1 \otimes_T G_2) = [14M_2(G_1) + 3M_3(G_1) + M_4(G_1) + 2\{\sum_{w,w'\in V(G_1)}\gamma_{ww'}\delta_{G_1}(w)\delta_{G_1}(w')$$
$$+\sum_{w'\in V(G_1)}\left(\delta_{G_1}(w')\right)^2\sum_{w\in V(G_1)s.t.\ ww'\in E(G_1)}\delta_{G_1}(w)\}]M_2(G_2),$$

where $\gamma_{ww'}$ *is the number of common neighbors of* w *and* w' *in* G_1.

Theorem 2.4: [7] *Let* G_1 *and* G_2 *be simple connected undirected graphs. Then, the forgotten index of* $G_1 \otimes_F G_2$ *are given as below:*

a. $M_3(G_1 \otimes_S G_2) = (M_3(G_1) + 8m_1)M_3(G_2)$.
b. $M_3(G_1 \otimes_R G_2) = 8(M_3(G_1) + m_1)M_3(G_2)$.
c. $M_3(G_1 \otimes_Q G_2) =$
$\left[M_3(G_1) + M_4(G_1) + 3\sum_{ww'\in E(G_1)}\delta_{G_1}(w)\delta_{G_1}(w')\left(\delta_{G_1}(w) + \delta_{G_1}(w')\right)\right]M_3(G_2)$.
d. $M_3(G_1 \otimes_T G_2) =$
$\left[8M_3(G_1) + M_4(G_1) + 3\sum_{ww'\in E(G_1)}\delta_{G_1}(w)\delta_{G_1}(w')\left(\delta_{G_1}(w) + \delta_{G_1}(w')\right)\right]M_3(G_2)$.

III. MAIN RESULTS

In this section, we deduce the expressions for the Reduced first Zagreb index, Reduced second Zagreb index and the hyper-Zagreb index of the new graphs. We use the expression for the difference of the Zagreb indices and proposition 2 from [13] to establish *theorems 3.1* to *3.8*. To establish *theorems 3.9* to *3.12* we use $HZ(G) = M_3(G) + 2M_2(G)$ from [12].

Theorem 3.1: *Let* G_i*; i=1,2 be a simple connected undirected graph with* $|V(G_i)| = n_i$ *and* $|E(G_i)| = m_i$. *Then,*
$RM_1(G_1 \otimes_S G_2) = M_1(G_1)M_1(G_2) + 4m_1M_1(G_2) + (m_1 + n_1)n_2 - 16m_1m_2$.

Theorem 3.2: *Let* G_i*; i=1,2 be a simple connected undirected graph with* $|V(G_i)| = n_i$ *and* $|E(G_i)| = m_i$. *Then,*
$RM_1(G_1 \otimes_R G_2) = 4M_1(G_1)M_1(G_2) + 4m_1M_1(G_2) + (m_1 + n_1)n_2 - 24m_1m_2$.

Theorem 3.3: *Let* G_i*; i=1,2 be a simple connected undirected graph with* $|V(G_i)| = n_i$ *and* $|E(G_i)| = m_i$. *Then,*
$RM_1(G_1 \otimes_Q G_2) = \{M_3(G_1) + 2M_2(G_1)\}M_1(G_2) + (m_1 + n_1)n_2 - \{8m_1 + 4M_1(G_1)\}m_2$.

Theorem 3.4: *Let* G_i*; i=1,2 be a simple connected undirected graph with* $|V(G_i)| = n_i$ *and* $|E(G_i)| = m_i$. *Then,*

$RM_1(G_1 \otimes_T G_2) = \{4M_1(G_1) + M_3(G_1) + 2M_2(G_1)\}M_1(G_2) + (m_1 + n_1)n_2 - \{16m_1 + 4M_1(G_1)\}m_2.$

Theorem 3.5: *Let G_i; i=1,2 be a simple connected undirected graph with $|V(G_i)| = n_i$ and $|E(G_i)| = m_i$. Then,*

$$RM_2(G_1 \otimes_S G_2) = M_1(G_1)\{4M_2(G_2) - M_1(G_2)\} - 4m_1M_1(G_2) + 4m_1m_2.$$

Theorem 3.6: *Let G_i; i=1,2 be a simple connected undirected graph with $|V(G_i)| = n_i$ and $|E(G_i)| = m_i$. Then,*

$$RM_2(G_1 \otimes_R G_2) = 4M_1(G_1)\{2M_2(G_2) - M_1(G_2)\} + 4m_1M_1(G_2) + 8M_2(G_1)M_2(G_2) + 6m_1m_2.$$

Theorem 3.7: *Let G_i; i=1,2 be a simple connected undirected graph with $|V(G_i)| = n_i$ and $|E(G_i)| = m_i$. Then,*

$$RM_2(G_1 \otimes_Q G_2) = \{2M_2(G_1) + M_3(G_1)\}\{M_2(G_2) - M_1(G_2)\} + [M_4(G_1) + 2\sum_{w,w' \in V(G_1)} \gamma_{ww'}\delta_{G_1}(w)\delta_{G_1}(w') + 2\sum_{w' \in V(G_1)} \left(\delta_{G_1}(w')\right)^2 \sum_{w \in V(G_1)\, s.t. ww' \in E(G_1)} \delta_{G_1}(w)]M_2(G_2) - M_1(G_1)M_1(G_2) + \{2m_1 + M_1(G_1)\}m_2.$$

Where $\gamma_{ww'}$ is the number common neighbor of w *and* w' in $V(G_1)$.

Theorem 3.8: : *Let G_i; i=1,2 be a simple connected undirected graph with $|V(G_i)| = n_i$ and $|E(G_i)| = m_i$. Then,*

$$RM_2(G_1 \otimes_T G_2) = 2M_2(G_1)\{7M_2(G_1)M_2(G_2) - M_1(G_2)\} + M_3(G_1)\{3M_2(G_2) - M_1(G_2)\} + \left[M_4(G_1) + 2\sum_{w,w' \in V(G_1)} \gamma_{ww'}\delta_{G_1}(w)\delta_{G_1}(w') + 2\sum_{w' \in V(G_1)} \left(\delta_{G_1}(w')\right)^2 \sum_{w \in V(G_1)\, s.t. ww' \in E(G_1)} \delta_{G_1}(w)\right]M_2(G_2) - 4M_1(G_1)M_1(G_2) + \{4m_1 + M_1(G_1)\}m_2.$$

Where $\gamma_{ww'}$ is the number common neighbor of w *and* w' in $V(G_1)$.

Proof of theorem 3.8:

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$$RM_2(G_1 \otimes_T G_2) = M_2(G_1 \otimes_T G_2) - M_1(G_1 \otimes_T G_2) + \{4m_1 + M_1(G_1)\}m_2$$
$$= \Bigg[14M_2(G_1) + 3M_3(G_1) + M_4(G_1) + \Bigg\{2 \sum_{w,w' \in V(G_1)} \gamma_{ww'} \delta_{G_1}(w)\delta_{G_1}(w') + 2\sum_{w' \in V(G_1)} \left(\delta_{G_1}(w')\right)^2 \sum_{w \in V(G_1)\, s.t. ww' \in E(G_1)} \delta_{G_1}(w)\Bigg\}\Bigg] M_2(G_2) - \{4M_1(G_1) + 2M_2(G_1) + M_3(G_1)\}M_1(G_2) + \{4m_1 + M_1(G_1)\}m_2$$
$$= 2M_2(G_1)\{7M_2(G_1)M_2(G_2) - M_1(G_2)\} + M_3(G_1)\{3M_2(G_2) - M_1(G_2)\} + \Bigg[M_4(G_1) + \Bigg\{2 \sum_{w,w' \in V(G_1)} \gamma_{ww'} \delta_{G_1}(w)\delta_{G_1}(w') + 2\sum_{w' \in V(G_1)} \left(\delta_{G_1}(w')\right)^2 \sum_{w \in V(G_1)\, s.t. ww' \in E(G_1)} \delta_{G_1}(w)\Bigg\}\Bigg] M_2(G_2) - 4M_1(G_1)M_1(G_2) + \{4m_1 + M_1(G_1)\}m_2. \square$$

Theorem 3.9: *Let G_i; i=1,2 be a simple connected undirected graph with $|V(G_i)| = n_i$ and $|E(G_i)| = m_i$. Then,*
$$HZ(G_1 \otimes_S G_2) = 8M_1(G_1)M_2(G_2) + \{M_3(G_1) + 8m_1\}M_3(G_2).$$

Theorem 3.10: *Let G_i; i=1,2 be a simple connected undirected graph with $|V(G_i)| = n_i$ and $|E(G_i)| = m_i$. Then,*
$$HZ(G_1 \otimes_R G_2) = 16\{M_1(G_1) + M_2(G_1)\}M_2(G_2) + 8\{M_3(G_1) + m_1\}M_3(G_2).$$

Theorem 3.11: *Let G_i; i=1,2 be a simple connected undirected graph with $|V(G_i)| = n_i$ and $|E(G_i)| = m_i$. Then,*
$$HZ(G_1 \otimes_Q G_2) = \{M_3(G_1) + M_4(G_1)\}M_3(G_2) + 2\{2M_2(G_1) + M_3(G_1) + M_4(G_1)\}M_2(G_2)$$
$$+3M_3(G_2) \sum_{ww' \in E(G_1)} \delta_{G_1}(w)\delta_{G_1}(w')\left(\delta_{G_1}(w) + \delta_{G_1}(w')\right)$$
$$+4M_2(G_2) \sum_{w' \in V(G_1)} \left(\delta_{G_1}(w')\right)^2 \sum_{w \in V(G_1)\, s.t. ww' \in E(G_1)} \delta_{G_1}(w).$$

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Theorem 3.12: *Let G_i; i=1,2 be a simple connected undirected graph with $|V(G_i)| = n_i$ and $|E(G_i)| = m_i$. Then,*

$$HZ(G_1 \otimes_T G_2) = \{8M_3(G_1) + M_4(G_1)\}M_3(G_2) + 2M_2(G_2)\{14M_2(G_1) + 3M_3(G_1) + M_4(G_1)\}M_2(G_2) + 3M_3(G_2) \sum_{ww' \in E(G_1)} \delta_{G_1}(w)\delta_{G_1}(w')(\delta_{G_1}(w) + \delta_{G_1}(w'))$$
$$+4\left\{ \sum_{w,w' \in V(G_1)} \gamma_{ww'} \delta_{G_1}(w)\delta_{G_1}(w') + \sum_{w' \in V(G_1)} \left(\delta_{G_1}(w')\right)^2 \sum_{w \in V(G_1)\ s.t.\ ww' \in E(G_1)} \delta_{G_1}(w) \right\} M_2(G_2).$$

Where $\gamma_{ww'}$ is the number common neighbor of w and w' in $V(G_1)$.

<u>Proof of theorem 3.12:</u>

$$HZ(G_1 \otimes_T G_2) = M_3(G_1 \otimes_T G_2) + 2M_2(G_1 \otimes_T G_2)$$
$$= \left\{ 8M_3(G_1) + M_4(G_1) + 3 \sum_{ww' \in E(G_1)} \delta_{G_1}(w)\delta_{G_1}(w')\left(\delta_{G_1}(w) + \delta_{G_1}(w')\right) \right\} M_3(G_2)$$
$$+ 2\left[14M_2(G_1) + 3M_3(G_1) + M_4(G_1) + \left\{ 2 \sum_{w,w' \in V(G_1)} \gamma_{ww'} \delta_{G_1}(w)\delta_{G_1}(w') + 2 \sum_{w' \in V(G_1)} \left(\delta_{G_1}(w')\right)^2 \sum_{w \in V(G_1)\ s.t.\ ww' \in E(G_1)} \delta_{G_1}(w) \right\} \right] M_2(G_2)$$
$$= \{8M_3(G_1) + M_4(G_1)\}M_3(G_2) + 3M_3(G_2) \sum_{ww' \in E(G_1)} \delta_{G_1}(w)\delta_{G_1}(w')\left(\delta_{G_1}(w) + \delta_{G_1}(w')\right)$$
$$+ 2M_2(G_2)\{14M_2(G_1) + 3M_3(G_1) + M_4(G_1)\}M_2(G_2)$$
$$+ 4\left\{ \sum_{w,w' \in V(G_1)} \gamma_{ww'} \delta_{G_1}(w)\delta_{G_1}(w') + \sum_{w' \in V(G_1)} \left(\delta_{G_1}(w')\right)^2 \sum_{w \in V(G_1)\ s.t.\ ww' \in E(G_1)} \delta_{G_1}(w) \right\} M_2(G_2). \ \square$$

IV. CONCLUSION

In this communication, we determine explicit expressions for the Reduced first Zagreb index, Reduced second Zagreb index and the hyper-Zagreb index of four new operations based on the tensor product of graphs. It will be interesting to compute newer topological indices based on this new operation of graphs.

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